CHAPTER 35 Light

Answers to Understanding the Concepts Questions

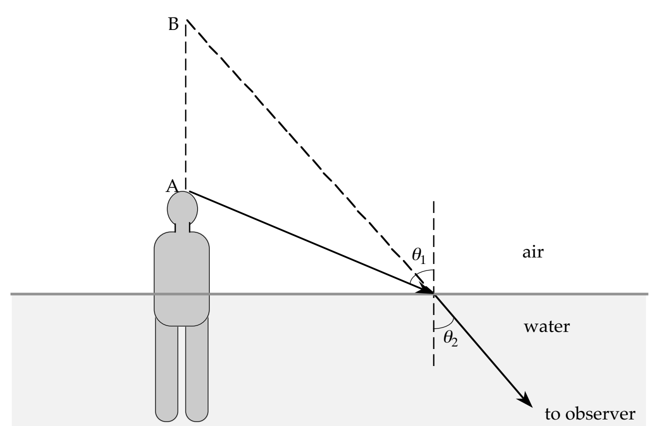
1. Light propagates within your house or other spaces mainly through multiple reflections from all the surfaces that are present. The wave-like bending around corners plays virtually no role at household scales.

2. The answer is (e) — none of these. The frequency is determined by the source of the light and therefore cannot change. Since the light reflects back into the same medium its speed is also unchanged, and neither can its wavelength, which is determined by its frequency and speed.

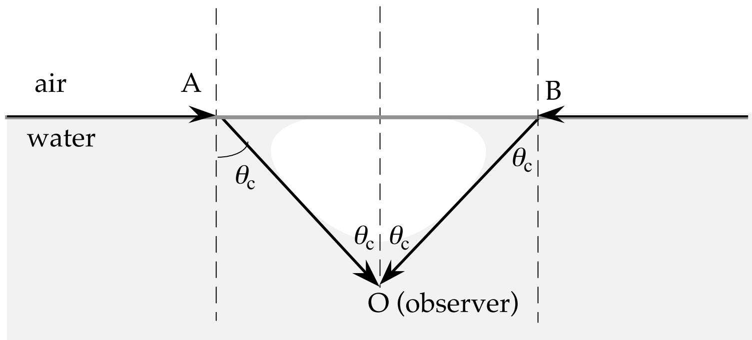
3. If the direction vector of an incident beam is $\vec{n} = n_x\hat{i} + n_y\hat{j} + n_z\hat{k}$, then as the beam reflects from a mirror in the xy plane, n_x and n_y remain the same while n_z reverses to $-n_z$. If only two perpendicular mirrors are used, say one in the xy plane and the other in the xz plane, then as the light beam gets reflected from them only n_y and n_z are reversed. To completely reverse the direction of the beam upon reflection, i.e., to make the direction vector of the outgoing beam $-\vec{n}$, we must make sure that $n_x = 0$, i.e., the incident beam must be parallel to the yz plane. This is the difficulty involved. But if three mutually perpendicular mirrors are used then all three components of $\vec{n}$ would reverse upon reflection (unless they are zero to begin with), so $\vec{n}$ always becomes $-\vec{n}$ — the reflected beam is always anti-parallel to the incident one.

4. A fish can see a fisherman before the fisherman can see the fish. This has nothing to do with the optical properties of their respective eyes or brains, but rather with the fact that water has a higher index of refraction than air, so that shallow rays from a fish towards a fisherman undergo total internal reflection rather than reaching the fisherman's eyes. On the other hand, all the rays from the fisherman can arrive at the eyes of the fish. We hope no fish are insulted by our insinuation that they cannot think.

5. It is clear from the figure shown here that the bodyguard would appear taller than his/her actual size to an underwater observer. The light ray illustrated that emerges from the head of the bodyguard (point A) makes an angle θ_1 with the normal, and it bends toward the normal after enter the water, making a smaller angle θ_2 with the normal. So it appears to the observer that the head of the bodyguard is at point B, which is above point A.



6. In the diagram below, a beam of light is incident from the air upon the air-water boundary at 90º from the normal, and the refracted beam makes an angle θ_c with the normal, where according to Snell's law $\sin\theta_c / \sin 90^\circ = n_{air} / n_{water} = 1/1.33$, or , or $\theta_c = \sin^{-1}(1/1.33) = 48.8^\circ$. Any ray from the air with an incident angle less than 90º would reach the observer with an angle of refraction less than θ_c. Thus the underwater observer can only see the outside world from a light cone of base diameter AB. The wall of the cone makes an angle of about 49º with the vertical, as calculated above.



7. First, the depth of the fish appears to be less than its true depth, because the rays from the fish are bent towards the horizontal as the leave the water. The fish also appears shorter, because the angle between rays from the head and the tail decreases. An easy way to see this is to think of the limiting case where the rays from the fish are almost at the critical angle for total internal reflection; in that case both rays are very close to the horizontal and the fish is seen highly shortened and only a little below the surface.

8. The angular frequency ω is determined by the source of the wave and therefore remains the same. The speed of the wave changes to $v = c/n$ in the medium, so k becomes $k' = \omega/v = \omega/(c/n) = n(\omega/c) = nk$.

9. The answer is (d). The frequency is determined by the source of the wave and therefore remains the same.

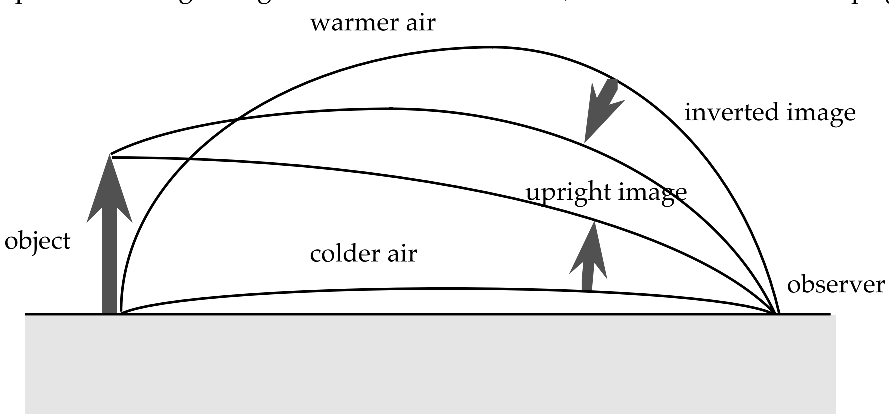
10. The changing color of the Sun has to do with the amount of atmosphere its rays pass through, which increases as the Sun falls to the horizon. The atmosphere scatters the shorter wavelengths more strongly than the longer wavelengths, leaving the longer wavelengths a more predominant part of the Sun's light as it approaches the horizon. The squashing has to do with increased refraction of the rays that travel through more of the atmosphere; that is, those from the bottom part of the Sun. Those rays are refracted more strongly towards Earth's surface, so that the eye interprets them as coming from a position higher up, i.e., closer to the geometric center of the Sun's disk. The form the disk takes is accordingly shortened at the bottom, or flattened.

11. Take the diagram above for Question 6, and imagine a coin lying at the location of the observer in that diagram. Reverse all the direction of the light rays to depict the light from the coin. Even though only the light rays within the light cone of base diameter AB can reach into the air, once in air these light rays are no longer concentrated within a cone; they can make an angle with the vertical anywhere between 0º and 90º — so they fill the entire airspace above the water surface. As long as you tilt your head towards the general direction of the coin for your eyes to catch these light rays, you should always be able to see the coin. (It will, however, appear to get closer and closer to the water surface as you move further away from it.)

12. Think of a series of horizontal layers of air, ever hotter as you approach the ground. Light from, say, the frond of the palm tree penetrates one of those layers and each time it does so it bends more away from the perpendicular to the horizontal boundary layer. This is characteristic of the lower (hotter)

layer having a smaller index of refraction than the upper (cooler) layer. Thus we conclude that light travels faster in hotter air. This conclusion is in accord with our understanding of; say, hot air balloons (see Chapter 16), in which we found that hot air is less dense than cool air. Since the index of refraction has its origin in rescattering from atoms, we would certainly expect that a system with a lower density of rescattering atoms has a lower index of refraction than one with more rescattering atoms per unit volume. You might want to think about how, once the ray is moving horizontally, it starts to move upward again. The answer to that has to do with total internal reflection.

13. Colder air is denser so its has a higher value of index of refraction than hotter air. Sometimes the air near the surface is colder. This can occur, for example, over the ocean when the water is cold. Light rays would bend downwards as they reach from colder to warmer air. The schematicray diagram below depicts *two* mirage images under such conditions, one inverted and one upright.



A famous mirage of this category, known as the Fatta Morgana, can be seen in the Strait of Messina, between Italy and Sicily. Here the distorted images or houses of the opposite cliffs are transformed into imaginary castles in sea and sky. The Italians name this mirage after the Fatta Morgana, legendary enchantress with the magical power of raising phantom castle from the waters. It may also be the cause of legends about phantom ships that sail the sky. Reports of the ghost ship Flying Dutchman may well have been the reflection of some distant vessel.

14. Since light travels slower in water than in air it would take more time for light to reflect back if it travels through a pipe filled with water. The speed of rotation of the wheel must be slowed down in order to make it work. If, instead, light were to travel faster in water then the effect would be just the opposite; and the wheel must be speeded up.

15. The blue sky is the result of the scattering of sunlight back to the eye by atmospheric molecules. In the vacuum of outer space, there is no scattering; light from the Sun that is not directed towards an astronaut cannot scatter from some other part of the sky to be redirected to the astronaut, and the sky appears dark.

16. Be definition the index of refraction of a certain medium is $n = c/v$, where c is the speed of light in vacuum and v is the speed of light in the medium. In vacuum $v = c$, so $n = 1$ (exactly).

17. This is best answered by drawing a series of light rays — in reverse — from your eye and, starting with a vertical line, moving down in angle. (The light actually comes from the lifeguard, of course, but we can certainly think about lines from your eye that follow the rays from the lifeguard.) As the line moves down you will encounter the lifeguard's head. It looks higher than it really is, because the ray from the lifeguard is bent to the vertical as it enters the water. When the line from your eye is at the critical angle for total internal reflection, the line grazes the surface outside; the corresponding ray comes from the lifeguard's waist. As the line drops further, you receive light from below the

lifeguard's waist that has reflected from the water surface and goes to your eye having reflected once. You see a section of the lower half of the lifeguard as if in a mirror, and that part will be upside down. Finally, the angle of the line from your eye drops enough so that it runs directly to the lifeguard's waist, and from that point on you see the entire bottom half of the lifeguard in its correct proportion and orientation.

18. If the laser beam strikes the boundary between the first rod and the air outside at an angle of incidence greater than the critical angle for total internal reflection, which satisfies sin $\theta_c = n_{air}/n_{glass}$, then total internal reflection occurs and no light is transmitted out of the first glass rod. If the rod is coated with glycerin, then we need to repace n_{air} with the index of refraction of glycerin, which is 1.473 instead of 1.00 (for air). This greatly increases the value of the critical angle for total internal reflection, which is now greater than the angle of incidence. No total internal reflection occurs, and the laser beam can now enter the second rod.

19. If the two surfaces of the glass are parallel, then it is not hard to show that any ray that enters the glass at some angle of incidence leaves it at exactly the same angle. However, the ray is displaced from the entering line by an amount that increases as the index of refraction increases. (Problem 32 treats this quantitatively.) If there is dispersion, then the amount of displacement does depend on the color. However, the angle with which the rays of different colors leave the glass is the same; that is, they make a parallel bundle.

20. It is straightforward to draw a ray diagram to see that everything submerged underwater appears to be vertically shorter to an observer in air — the exact opposite to the case discussed in Question 5, where a person in air appears to be taller to an underwater observer. Therefore, the submerged pin appears shorter when viewd in air (i.e, it does not appear to stick out of the bottom of the cork as much as it actually does). As a result it may not be visible due to the obstruction of the cork.

21. The setting Sun is red as the atmosphere scatters away the shorter wavelength components (such as blue and green) of the sunlight, making the Sun appear red. The Moon does not produce light itself, and the sunlight reflected from the Moon has relatively little red component, so the Moon generally does not appear red (even though under certain weather conditions, such as in the desert, it does sometimes appear reddish around sunset.)

Solutions to Problems

1. We find the speed of light from the index of refraction, $v = c/n$:

   $v_{\text{ice}} = (3.00 \times 10^8 \text{ m/s})/1.31 = \boxed{2.29 \times 10^8 \text{ m/s}}$;

   $v_{\text{ethyl alcohol}} = (3.00 \times 10^8 \text{ m/s})/1.36 = \boxed{2.21 \times 10^8 \text{ m/s}}$;

   $v_{\text{benzene}} = (3.00 \times 10^8 \text{ m/s})/1.50 = \boxed{2.00 \times 10^8 \text{ m/s}}$;

   $v_{\text{diamond}} = (3.00 \times 10^8 \text{ m/s})/2.42 = \boxed{1.24 \times 10^8 \text{ m/s}}$.

2. We let units help us convert the distance:

   $d = ct = (3.0 \times 10^8 \text{ m/s})(4.2 \text{ yr})(365 \text{ d/yr})(24 \text{ h/d})(3600 \text{ s/h}) = \boxed{4.0 \times 10^{16} \text{ m}}$.

3. We find the wavelength from

   $\lambda_1 = \lambda_0/n_1 = (650 \text{ nm})/1.32 = \boxed{492 \text{ nm}}$.

   The frequency does not change:

   $f_1 = f_0 = c/\lambda_0 = (3.00 \times 10^8 \text{ m/s})/(650 \times 10^{-9} \text{ m}) = \boxed{4.62 \times 10^{14} \text{ Hz}}$.

4. We find the wavelengths from

   $\lambda_0 = c/f = (3.00 \times 10^8 \text{ m/s})/(5.6 \times 10^{14} \text{ Hz}) = 0.536 \times 10^{-6} \text{ m} = \boxed{536 \text{nm}}$.

   $\lambda_{\text{glass}} = v_{\text{glass}}/f = c/n_{\text{glass}}f = \lambda_0/n_{\text{glass}} = (536 \text{ nm})/(1.45) = \boxed{369 \text{ nm}}$.

   We find the index of refraction of the material from

   $\lambda_1 = \lambda_0/n_1 = \lambda_0/2$, which gives $n_1 = \boxed{2.0}$.

5. The transit time of the light beam must equal the time for the circumference of the wheel to travel the distance between openings:

   $\Delta t = 2D/c = L/R\omega$;

   $(1000 \text{ m})/(3.0 \times 10^8 \text{ m/s}) = (1.5 \times 10^{-3} \text{ m})/(15.0 \times 10^{-2} \text{ m})\omega$, which gives

   $\omega = 3.0 \times 10^3 \text{ rad/s}$, or $f = \boxed{2.9 \times 10^4 \text{ rev/min}}$.

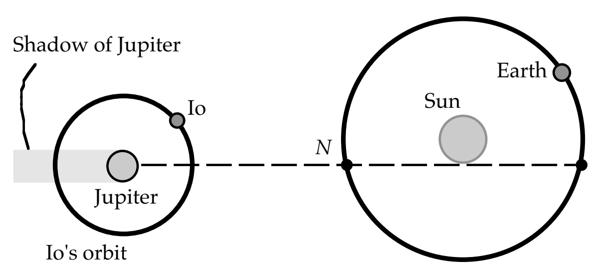
   This is an extremely high rotation rate. The wheel must have the material strength to provide the necessary centripetal force to prevent the wheel from flying apart.

6. The eclipse will be seen later because the light must travel the diameter of the earth's orbit:

   $\Delta t = 2R_{\text{earth-sun}}/c$

   $= 2(1.50 \times 10^{11} \text{ m})/(3.00 \times 10^8 \text{ m/s})$

   $= 1.00 \times 10^3 \text{ s} = \boxed{16.7 \text{ min}}$.



7. We find the distance between pulses from

   $d = c\,\Delta t = (3 \times 10^8 \text{ m/s})/(5 \times 10^7 \text{ s}^{-1}) = \boxed{6 \text{m}}$.

   This is a relatively long distance; however, as the rate increases, it decreases and may approach the size of a pulse where synchronization problems may arise unless the size of the circuit is reduced.

8. The time to travel by cable is

   $t_{\text{cable}} = D_{\text{cable}}/c = (10000 \times 10^3 \text{ m})/(3.0 \times 10^8 \text{ m/s}) = \boxed{0.034 \text{s}}$.

   The time to travel to and from the satellite is

   $t_{\text{satellite}} = 2D_{\text{satellite}}/c \simeq 2(40{,}600 \text{ km} - 6400 \text{ km})(10^3 \text{ m/km})/(3.0 \times 10^8 \text{ m/s}) = \boxed{0.23 \text{ s}}$.

   In communication this is a noticeable difference.

9. The smallest time interval determines the uncertainty in the distance being measured:

   $\Delta t_{min} = 2\,\Delta x/c$

   $= 2(15 \times 10^{-2}\ \text{m})/(2.998 \times 10^{8}\ \text{m/s}) = 1.0 \times 10^{-9}\ \text{s} = \boxed{1.0\ \text{ns}}$.

10. We estimate the reaction time for someone who is prepared to react as $\frac{3}{4}$ s. Because each person must react to a signal, we have

    $\Delta t \simeq 2t_{reaction} \simeq 2(0.75\ \text{s}) \simeq \boxed{1.5\ \text{s}}$.

    The transit time for the light is

    $\Delta t_{light} = 2L/c = 2(4 \times 10^{3}\ \text{m})/(3 \times 10^{8}\ \text{m/s}) = \boxed{3 \times 10^{-5}\ \text{s}}$.

11. The rate of rotation of the wheel is

    $\omega = (1.2 \times 10^{5}\ \text{rev/min})(2\pi\ \text{rad/rev})/(60\ \text{s/min}) = 1.26 \times 10^{4}\ \text{rad/s}$.

    The transit time of the light beam must equal the time for the circumference of the wheel to travel the distance between openings:

    $\Delta t = 2D/c = L/R\omega$;

    $2(1500\ \text{m})/(3 \times 10^{8}\ \text{m/s}) = L/(10 \times 10^{-2}\ \text{m})(1.26 \times 10^{4}\ \text{rad/s})$, which gives $L = 1.3 \times 10^{-2}\ \text{m} = \boxed{1.3\ \text{cm}}$.

12. For reflection, the angle of incidence is equal to the angle of reflection. The angle the beam turns is $180° - (\theta + \theta')$. To turn the beam by 75°, the sum of these two angles must be 115°:

    $\theta = \theta'$, $\theta + \theta' = 115°$.

    Therefore, the angle between the beam and the mirror is half of 75°, or $\boxed{37.5°}$.

13. When the light in the material is incident at the critical angle, the refracted angle in air is 90°:

    $n_1 \sin\theta_1 = n_2 \sin\theta_2$;

    $n_1 \sin 38° = (1.00) \sin 90°$, which gives $n_1 = \boxed{1.62}$.

14. If θ is the angle between the incident beam and the normal to the mirror, the rate of change of this angle is the rate of rotation of the mirror:

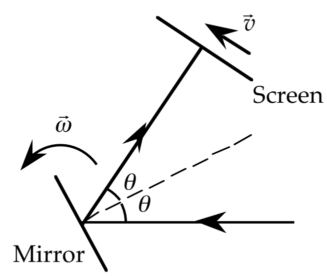
    $d\theta/dt = \omega$.

    The angle between the incident beam and the reflected beam is 2θ.
    The tangential speed of the reflected beam on the screen is

    $v = R\,d(2\theta)/dt = 2R\,d\theta/dt = 2R\omega$

    $= 2(20\ \text{m})(30\ \text{rev/min})(2\pi\ \text{rad/rev})/(60\ \text{s/min})$

    $= \boxed{1.3 \times 10^{2}\ \text{m/s}}$.



15. We find the angle of refraction in the water from

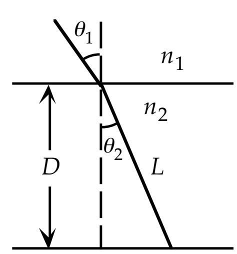
    $n_1 \sin\theta_1 = n_2 \sin\theta_2$;

    $(1.00) \sin 45° = (1.33) \sin\theta_2$, which gives $\theta_2 = 32°$.

    We find the distance the light travels from

    $\cos\theta_2 = D/L$;

    $\cos 32° = (500\ \text{m})/L$, which gives $L = \boxed{590\ \text{m}}$.



16. We find the angle of refraction from

    $n_1 \sin\theta_1 = n_2 \sin\theta_2$, or $\sin\theta_2 = (n_1/n_2)\sin\theta_1$.

    Because $n_2 > n_1$, $\sin\theta_2 < \sin\theta_1$, or $\theta_2 < \theta_1$. The maximum incident angle is 90°, so the maximum angle of refraction must be less than 90°. We find the maximum angle of refraction from

    $\sin\theta_{2max} = n_1/n_2 = 1.33/1.50$, which gives $\theta_{2max} = \boxed{62.5°}$.

17. We find the fraction of light reflected from

$$I_r/I_0 = (n_2 - n_1)^2/(n_2 + n_1)^2 = (1.43 - 1)^2/(1.43 + 1)^2 = \boxed{0.031}.$$

Note that, if there is little light coming from inside the store, this will be enough to see a reflected image.

18. The light from directly overhead will travel straight to her eyes. Light from the horizon will be incident at an angle of 90° and be refracted to her eyes:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 ;$$

$$(1.00) \sin 90° = (1.33) \sin \theta_2 \text{, which gives } \theta_2 = 49°.$$

Thus she must move her eyes through an angle of $\boxed{98°}$ to see across the whole sky.

19. We find the angle of refraction from

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 ;$$

$$(1.00) \sin 90° = (1 + 2.93 \times 10^{-4}) \sin \theta_2 \text{, which gives } \theta_2 = 88.6°.$$

The angle with the horizontal is

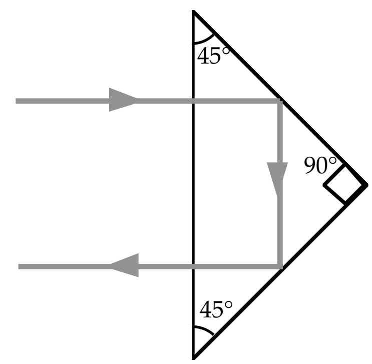
$$\phi = 90° - \theta_2 = \boxed{1.4°}.$$

20. When the light in the glass is incident at the critical angle, the refracted angle in air is 90°:

$$n_{\text{glass}} \sin \theta_c = n_{\text{air}} \sin \theta_{\text{air}};$$

$$(1.46) \sin \theta_c = (1.00) \sin 90°\text{, which gives } \theta_c = \boxed{43.2°.}$$

At the entrance, the ray is undeviated, so the angle of incidence at the second side is 45°, which is greater than the critical angle (41.1° for crown glass of n =1.52). The ray totally reflects at 45°, so the reflection is repeated at the next surface.The ray is then incident on the original side at 0°, so it is undeviated. The net effect is a reversal of direction, with no loss of light.



$\boxed{21}$. We find the angle in the water from the refraction at the air–water surface:

$$n_{\text{air}} \sin \theta_{\text{air}} = n_{\text{water}} \sin \theta_{\text{water}} ;$$

$$(1.00) \sin 72° = (1.33) \sin \theta_{\text{water}}\text{, which gives } \theta_{\text{water}} = \boxed{45.7°}.$$

Because the surfaces are parallel, this is the angle from the normal of the ray incident on the glass. We find the angle in the glass from the refraction at the water–glass surface:

$$n_{\text{water}} \sin \theta_{\text{water}} = n_{\text{air}} \sin \theta_{\text{air}} = n_{\text{glass}} \sin \theta_{\text{glass}};$$

$$(1.00) \sin 72° = (1.53) \sin \theta_{\text{glass}}\text{, which gives } \theta_{\text{glass}} = \boxed{38.5°}.$$

Note that θ_{glass} is independent of the presence of the water.

22. Because all of the surfaces are parallel, the angle of refraction from one surface is the angle of incidence at the next:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 ;$$

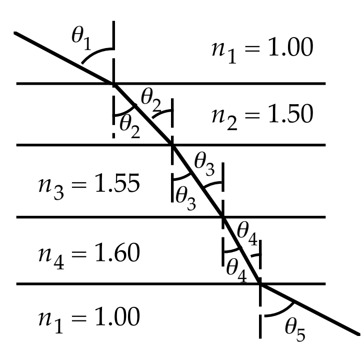
$$n_2 \sin \theta_2 = n_3 \sin \theta_3 ;$$

$$n_3 \sin \theta_3 = n_4 \sin \theta_4 ;$$

$$n_4 \sin \theta_4 = n_1 \sin \theta_5 , \quad \text{or}$$

$$n_1 \sin \theta_1 = n_1 \sin \theta_5 \text{, which gives } \quad \theta_5 = \theta_1 = 60°.$$

Because the ray emerges in the same index of refraction, it is undeviated.



23. When total internal reflection occurs, for the glass–air interface we have

$$n \sin \theta_i = n_{\text{air}} \sin \theta_{\text{air}} = \sin \theta_{\text{air}} > 1.$$

When we add the stack of layers, at the first layer we have

$$n \sin \theta_i = n_1 \sin \theta_1 .$$

If the value of n_1 is such that $\sin \theta_1 > 1$, total reflection will occur at this surface. If not, light will go to the next surface. At each surface, if total internal reflection does not occur, we continue until we get to air:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 ;$$
$$n_2 \sin \theta_2 = n_3 \sin \theta_3 ;$$
$$\vdots$$
$$n_k \sin \theta_k = n_{\text{air}} \sin \theta_{\text{air}} = \sin \theta_{\text{air}} ;$$ so, when we add all of the equations, we get

$n \sin \theta_i = \sin \theta_{\text{air}}$; which is the original condition, so total internal reflection must occur.

If total internal reflection does not occur at the glass–air interface, we have

$$n \sin \theta_i = n_{\text{air}} \sin \theta_{\text{air}} = \sin \theta_{\text{air}} < 1.$$

When we add the layers, at the first layer we have

$$n \sin \theta_i = n_1 \sin \theta_1 < 1.$$

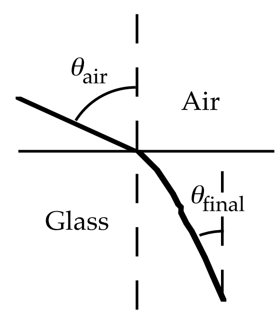
Because $n_1 > 1$, $\sin \theta_1 < 1$, so total reflection will not occur at this surface. The light will go to the next surface. The same reasoning shows that at each surface total internal reflection does not occur. As before, we continue until we get to air and add the equations to get

$n \sin \theta_i = \sin \theta_{\text{air}}$; which is the original condition, so total internal reflection does not occur.

The property of the interface $\boxed{\text{cannot be changed}}$ by adding a stack of layers.

24. We consider the glass block to consist of many thin layers, with each layer of constant index of refraction. If we say we have k layers, where k is very large, the index of refraction of the kth layer is

$$n_k = n(x) \text{ as } x \to \infty:$$
$$n_k = 1.54 - (0.18 \text{ cm}^2)/(\infty + 1 \text{ cm})^2 = 1.54.$$



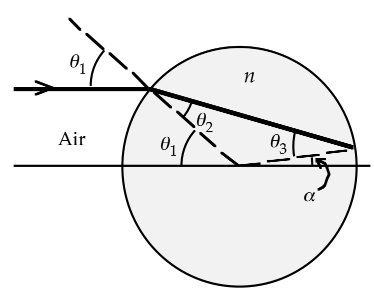
For the series of layers, we have

$$n_{\text{air}} \sin \theta_{\text{air}} = n_1 \sin \theta_1 ;$$
$$n_1 \sin \theta_1 = n_2 \sin \theta_2 ;$$
$$n_2 \sin \theta_2 = n_3 \sin \theta_3 ;$$
$$\vdots$$
$$n_{k-1} \sin \theta_{k-1} = n_k \sin \theta_k ;$$ so, when we add all of the equations, we get

$$n_{\text{air}} \sin \theta_{\text{air}} = n_k \sin \theta_k ;$$

$(1.00) \sin 55° = (1.54) \sin \theta_k$, which gives $\theta_k = \theta_{\text{final}} = \boxed{32.1°}$.

25. At the first surface, we have



$$n_{\text{air}} \sin \theta_1 = n \sin \theta_2;$$

$(1.00) \sin 40° = (1.6) \sin \theta_2$, which gives $\theta_2 = 23.7° = \theta_3$.

The ray strikes the back of the sphere at a latitude of

$$\alpha = 180° - [40° + (180° - 2(23.7°)] = \boxed{7.4°}.$$

From the isosceles triangle, we see that the angle of incidence at the back of the sphere is $\theta_3 = 23.7°$. The critical angle is

$$n \sin \theta_c = n_{\text{air}} \sin 90°;$$

$(1.6) \sin \theta_c = (1.00)(1)$, which gives $\theta_c = 38.7°$.

Because $\theta_3 < \theta_c$, $\boxed{\text{there will not be total internal reflection}}$.

26. For the blue light, we have

$n_{\text{air}} \sin \theta_1 = n_{\text{blue}} \sin \theta_{2\text{blue}}$;

$(1.00) \sin 25° = (1.528) \sin \theta_{2\text{blue}}$, which gives $\theta_{2\text{blue}} = 16.06°$.

We find the angle of incidence at the second surface from

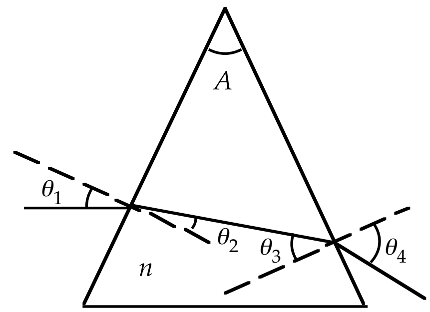
$(90° - \theta_{2\text{blue}}) + (90° - \theta_{3\text{blue}}) + A = 180°$, which gives

$\theta_{3\text{blue}} = A - \theta_{2\text{blue}} = 50° - 16.06° = 33.94°$.

For the refraction at the second surface, we have

$n_{\text{blue}} \sin \theta_{3\text{blue}} = n_{\text{air}} \sin \theta_{4\text{blue}}$;

$(1.528) \sin 33.94° = (1.00) \sin \theta_{4\text{blue}}$, which gives $\theta_{4\text{blue}} = 58.56°$.



For the red light, we have

$n_{\text{air}} \sin \theta_1 = n_{\text{red}} \sin \theta_{2\text{red}}$;

$(1.00) \sin 25° = (1.514) \sin \theta_{2\text{red}}$, which gives $\theta_{2\text{red}} = 16.21°$.

We find the angle of incidence at the second surface from

$\theta_{3\text{red}} = A - \theta_{2\text{red}} = 50° - 16.21° = 33.79°$.

For the refraction at the second surface, we have

$n_{\text{red}} \sin \theta_{3\text{red}} = n_{\text{air}} \sin \theta_{4\text{red}}$;

$(1.514) \sin 33.79° = (1.00) \sin \theta_{4\text{red}}$, which gives $\theta_{4\text{red}} = 57.36°$.

Because the screen is far away, with the screen perpendicular to the beam we find the separation from

$\Delta s = R\, \Delta\theta_4 = (10 \text{ m})(58.56° - 57.36°)(\pi \text{ rad}/180°) = 0.21 \text{ m} = \boxed{21 \text{ cm}}$.

27. Light that strikes the top half of the prism will be refracted downward. From the symmetry of the prism and its placement, light that strikes the bottom half will be refracted upward at the same angle. Thus we find what happens to the light passing through the top half and invert the result for the bottom half. At the first surface of the prism, we have

$n_{\text{air}} \sin \theta_1 = n \sin \theta_2$;

$(1.00) \sin 30° = (1.5) \sin \theta_2$, which gives $\theta_2 = 19.5°$.

We find the angle of incidence at the second surface from

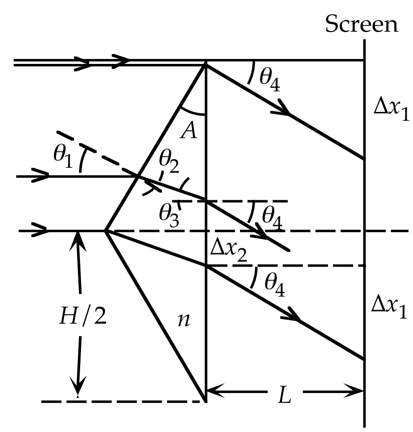
$(90° - \theta_2) + (90° - \theta_3) + A = 180°$, which gives

$\theta_3 = A - \theta_2 = 30° - 19.5° = 10.5°$.

For the refraction at the second surface, we have

$n \sin \theta_3 = n_{\text{air}} \sin \theta_4$;

$(1.5) \sin 10.5° = (1.00) \sin \theta_4$, which gives $\theta_4 = 15.9°$.



The ray that passes through the top edge of the prism will strike the screen a distance

$\Delta x_1 = L \tan \theta_4 = (2 \text{ cm}) \tan 15.9° = 0.57$ cm below the top.

The ray that passes just above the center of the prism will strike the second surface a distance

$\Delta x_2 = a \tan \theta_3 = \frac{1}{2}H \tan A \tan \theta_3 = \frac{1}{2}(3 \text{ cm}) \tan 30° \tan 10.5° = 0.16$ cm below the center.

This ray will strike the screen 0.57 cm below this point, which is

$\Delta x = \Delta x_1 + \Delta x_2 = 0.16 \text{ cm} + 0.57 \text{ cm} = 0.73$ cm below the center of the prism or

$\frac{1}{2}H - \Delta x = 1.5 \text{ cm} - 0.73 \text{ cm} = 0.77$ cm above the bottom edge of the prism.

From the symmetry of the pattern, we have the following:

Outside the "shadow" of the prism, there is direct illumination. For a distance of 0.57 cm from each edge, there will be no illumination. For the next distance of 0.77 cm – 0.57 cm = 0.20 cm, there will be illumination from one half. For the next distance of 0.73 cm to the center of the pattern, there will be illumination from both halves.

28. For the refraction from air into water, we have

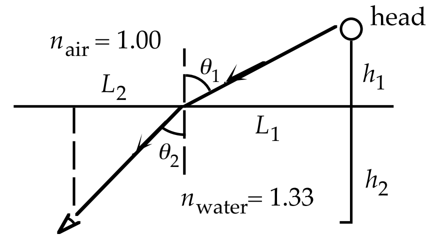
$n_{air} \sin \theta_1 = n_{water} \sin \theta_2$;

$(1.00) \sin \theta_1 = (1.33) \sin 46°$, which gives $\theta_1 = 73°$.

We find the horizontal distance from the eye to the feet from

$L = L_1 + L_2 = h_1 \tan \theta_1 + h_2 \tan \theta_2$

$= (0.90 \text{ m}) \tan 46° + (0.88 \text{ m}) \tan 73° = \boxed{3.81 \text{ m}}$.



29. For the refraction from water into air, we have

$n_{air} \sin \theta_1 = n_{water} \sin \theta_2$;

$(1.00) \sin \theta_1 = (1.33) \sin \theta_2$.

Both angles are unknown, so we use the distance along the glass:

$D \tan \theta_1 + D \tan \theta_2 = 2D/\tan 30° = 3.464$ m.

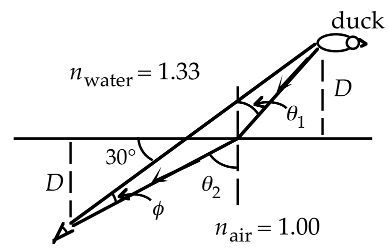
$\tan \theta_1 + \tan \theta_2 = 2/\tan 30° = 3.464$.

Because of the presence of the trigonometric functions, we solve the two equations numerically to get

$\theta_1 = 44.3°$ and $\theta_2 = 68.3°$.

The angle between the lines of sight is

$\phi = \theta_2 - \theta_1 = 68.3° - 44.3° = \boxed{8.3°}$.



30. In the diagram shown

$\sin \theta_1/\sin \theta = 1/n_1$, so

$d_1 = t_1 \tan \theta_1 = t_1 \sin \theta_1 / \cos \theta_1 = t_1 (\sin \theta / n_1)/[1 - (\sin \theta / n_1)^2]$.

Similarly,

$\sin \theta_2/\sin \theta_1 = n_1/n_2$, so $\sin \theta_2 = \sin \theta_1 (n_1/n_2) = \sin \theta (1/n_2)$ and

$d_2 = t_2 \tan \theta_2 = t_2 \sin \theta_2 / \cos \theta_2$

$= t_2 (\sin \theta / n_2)/[1 - (\sin \theta / n_2)^2]$.

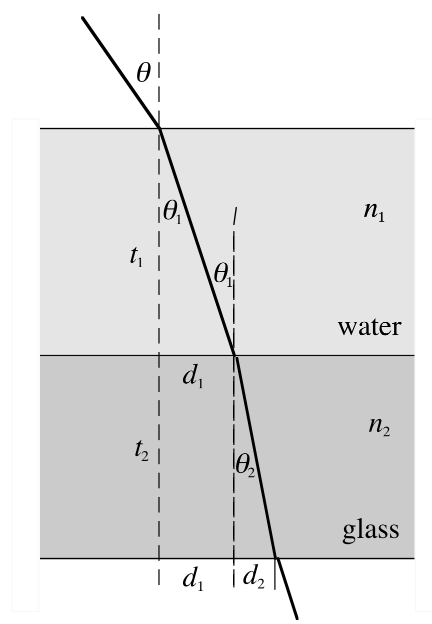
The total amount of horizontal shift of the beam is

$d = d_1 + d_2$

$= t_1 (\sin \theta / n_1)/[1 - (\sin \theta / n_1)^2] +$

$t_2 (\sin \theta / n_2)/[1 - (\sin \theta / n_2)^2]$.

Interchanging the water tank and the glass slab merely reverses the order of the two additive terms (d_1 and d_2) above, and leaves the result $\boxed{\text{unchanged}}$.



31. Refer to the diagram to the right. The flashlight is located at point A and its image is at A'. We have

$$OB = a \tan\theta = (15 \text{ cm}) \tan 60° = 26.0 \text{ cm}.$$

From Snell's law

$$\sin\theta_1/\sin\theta = 1/n_{\text{water}}\text{ , so}$$

$$\theta_1 = \sin^{-1}(\sin\theta/n_{\text{water}}) = \sin^{-1}(\sin 60°/1.6) = 32.8°;\text{ so}$$

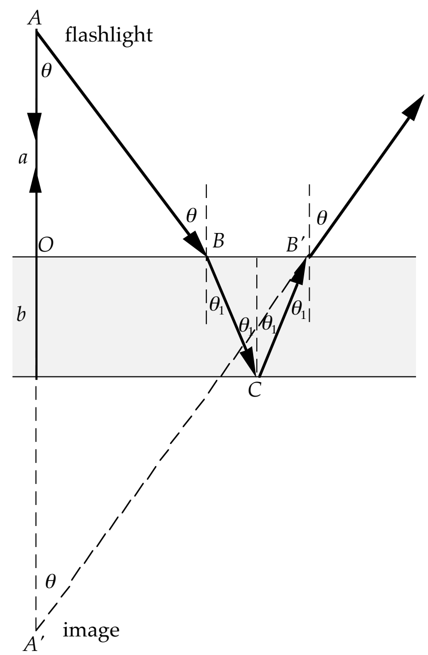
$$BB' = 2b \tan\theta_1 = 2(5 \text{ cm}) \tan 32.8° = 6.43 \text{ cm} \quad\text{and}$$

$$OB' = OB + BB' = 26.0 \text{ cm} + 6.43 \text{ cm} = 32.4 \text{ cm}.$$

In the right-angled triangle $OA'B'$, then,

$$OA' = OB'/\tan\theta$$
$$= (32.4 \text{ cm})/\tan 60°$$
$$= 18.7 \text{ cm} \approx 19 \text{ cm},$$

i.e., the image of the flashlight appears to be at point A', $\boxed{\text{19 cm below the upper surface of the glass}}$.



32. Because the glass surfaces are parallel, the exit beam will be traveling in the same direction as the original beam.
We find the angle inside the glass from

$$n_{\text{air}} \sin\theta_1 = n_{\text{glass}} \sin\theta_2;$$

$$(1.00) \sin 30° = (1.60) \sin\theta_2\text{, which gives } \theta_2 = 18.2°.$$

Relative to the entrance position, we find the distance along the glass where the exit beam leaves from

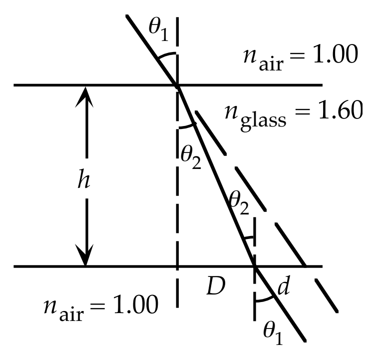
$$D = h \tan\theta_2 = (6 \text{ mm}) \tan 18.2° = 2.0 \text{ mm}.$$

The exit beam leaves 6 mm below and 2.0 mm along the glass and is parallel to the incident beam.
We find the displacement from where the incident beam would have been along the glass surface from

$$d = h \tan\theta_1 - h \tan\theta_2 = h(\tan\theta_1 - \tan\theta_2)$$
$$= (6 \text{ mm})(\tan 30° - \tan 18.2°) = \boxed{1.5 \text{ mm}}.$$

The perpendicular displacement from the original direction is

$$d \cos\theta_1 = (1.5 \text{ mm}) \cos 30° = 1.3 \text{ mm}.$$



$\boxed{33}$. For the refraction at the first surface, we have

$$n_{\text{air}} \sin\theta_1 = n \sin\theta_2;$$

$$(1.00) \sin 35° = (1.55) \sin\theta_2\text{, which gives } \theta_2 = 21.7°.$$

We find the angle of incidence at the second surface from

$$(90° - \theta_2) + (90° - \theta_3) + A = 180°\text{, which gives}$$

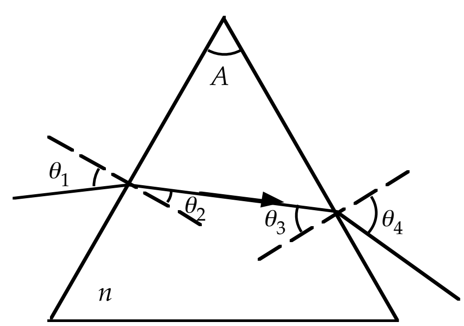
$$\theta_3 = A - \theta_2 = 60° - 21.7° = 38.3°.$$

For the refraction at the second surface, we have

$$n \sin\theta_3 = n_{\text{air}} \sin\theta_4;$$

$$(1.55) \sin 38.3° = (1.00) \sin\theta_4\text{, which gives}$$

$$\theta_4 = \boxed{74° \text{ from the normal}}.$$



34. (a) For the refraction at the side of the rod, we have

$n_2 \sin \theta_3 = n_1 \sin \theta_4$.

The minimum angle for total reflection $\theta_{3\text{min}}$ occurs when $\theta_4 = 90°$:

$(1.46) \sin \theta_{3\text{min}} = (1.00)(1)$, which gives $\theta_{3\text{min}} = 43.2°$.

We find the maximum angle of refraction at the end of the rod from

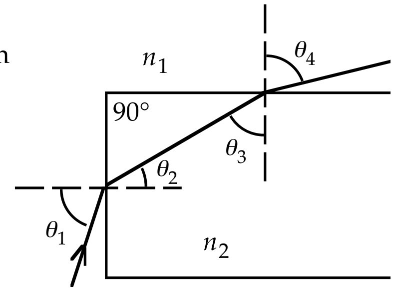
$\theta_{2\text{max}} = 90° - \theta_{3\text{min}} = 90° - 43.2° = 46.8°$.

Because the sine function increases with angle, for the refraction at the end of the rod, we have

$n_1 \sin \theta_{1\text{max}} = n_2 \sin \theta_{2\text{max}}$;

$(1.00) \sin \theta_{1\text{max}} = (1.46) \sin 46.8°$, which gives $\sin \theta_{1\text{max}} = 1.1$.

The maximum value of the sine function is 1, $\theta_{1\text{max}} = \boxed{90°}$, and total internal reflection will occur for all angles of incidence.



(b) The minimum angle for total reflection $\theta_{3\text{min}}$ occurs when $\theta_4 = 90°$:

$n_2 \sin \theta_3 = n_1 \sin \theta_4$;

$(1.46) \sin \theta_{3\text{min}} = (1.33)(1)$, which gives $\theta_{3\text{min}} = 65.6°$.

We find the maximum angle of refraction at the end of the rod from

$\theta_{2\text{max}} = 90° - \theta_{3\text{min}} = 90° - 65.6° = 24.4°$.

For the refraction at the end of the rod, we have

$n_1 \sin \theta_{1\text{max}} = n_2 \sin \theta_{2\text{max}}$;

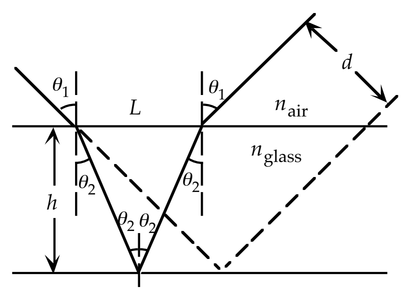
$(1.33) \sin \theta_{1\text{max}} = (1.46) \sin 24.4°$, which gives $\theta_{1\text{max}} = \boxed{27°}$.

All incident angles between 0° and 27° will internally reflect inside the rod.

35. We find the angle inside the glass from

$n_{\text{air}} \sin \theta_1 = n_{\text{glass}} \sin \theta_2$;

$(1.00) \sin 60° = (1.54) \sin \theta_2$, which gives $\theta_2 = 34.2°$.



For the reflection, the angle of incidence is equal to the angle of reflection. The refraction when the ray leaves the glass is the reverse of the initial refraction. Relative to the entrance position, we find the distance along the glass where the exit beam leaves from

$L_2 = 2h \tan \theta_2 = 2(5 \text{ mm}) \tan 34.2° = 6.8 \text{ mm}$.

We can use the same analysis to find the distance along the same line where the incident beam would have been without the glass, by setting $\theta_2 = \theta_1$:

$L_1 = 2h \tan \theta_1 = 2(5 \text{ mm}) \tan 60° = 17.3 \text{ mm}$.

The difference in the distance is

$D = L_1 - L_2 = 17.3 \text{ mm} - 6.8 \text{ mm} = 10.5 \text{ mm}$ along the glass surface.

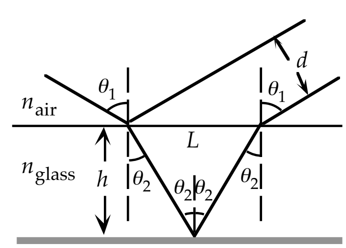
The displacement perpendicular to the direction of the reflected beam is

$d = D \cos \theta_1 = (10.5 \text{ mm}) \cos 60° = \boxed{5.3 \text{ mm}}$.

36. We find the angle inside the glass from

$n_{\text{air}} \sin \theta_1 = n_{\text{glass}} \sin \theta_2$;

$(1.00) \sin 70° = (1.45) \sin \theta_2$, which gives $\theta_2 = 40.4°$.



For the reflection, the angle of incidence is equal to the angle of reflection. The refraction when the ray leaves the glass is the reverse of the initial refraction. Relative to the entrance position, we find the distance along the glass where the exit beam leaves from

$L = 2h \tan \theta_2 = 2(2.0 \text{ mm}) \tan 40.4° = 3.4 \text{ mm}$.

The displacement perpendicular to the direction of the reflected beam is

$d = L \sin (90° - \theta_1) = (3.4 \text{ mm}) \sin 20° = \boxed{1.2 \text{ mm}}$.

$\boxed{37}$. We find the angle the light makes with the vertical from the length of the original shadow:

$D_1 = L \tan \theta_1$;

$1.0 \text{ m} = (2.0 \text{ m}) \tan \theta_1$, which gives $\theta_1 = 26.6°$.

We find the angle of the light in the water from

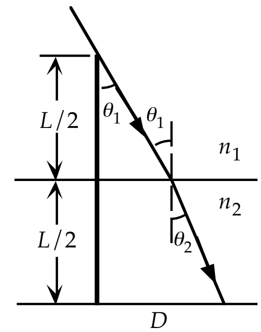
$n_1 \sin \theta_1 = n_2 \sin \theta_2$;

$(1.00) \sin 26.6° = (1.33) \sin \theta_2$, which gives $\theta_2 = 19.6°$.

The length of the shadow on the floor of the pool is

$D = \frac{1}{2}L \tan \theta_1 + \frac{1}{2}L \tan \theta_2 = \frac{1}{2}L(\tan \theta_1 + \tan \theta_2)$

$= \frac{1}{2}(2.0 \text{ m})(\tan 26.6° + \tan 19.6°) = \boxed{0.86 \text{ m}}$.



38. From the first experiment, we have

$n_1 \sin \theta_i = n_2 \sin \theta_j$;

$n_1 \sin 22° = n_2 \sin 29°$, which gives $n_1/n_2 = \boxed{1.295}$.

From the second experiment, we have

$n_2 \sin \theta_i = n_3 \sin \theta_j$;

$n_2 \sin 35° = n_3 \sin 53°$, which gives $n_2/n_3 = \boxed{1.355}$.

We find the third ratio from

$n_1/n_3 = (n_1/n_2)(n_2/n_3) = (1.295)(1.355) = \boxed{1.754}$.

39. At the first prism–dielectric surface, we have

$n_1 \sin \theta_1 = n_2 \sin \theta_2$;

$(1.814) \sin 45° = (1.380) \sin \theta_2$, which gives $\sin \theta_2 = 0.93$.

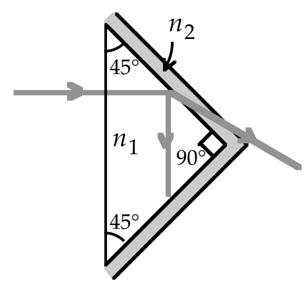
Because $\sin \theta_2 < 1$, $\boxed{\text{the prism will not be totally reflecting}}$; the light will exit at the first prism–dielectric surface.

To find the critical condition, we rearrange the refraction equation:

$n_2 = [(1.814) \sin 45°]/\sin \theta_2 = 1.283/\sin \theta_2$.

For the prism to be totally reflecting, we must have $\sin \theta_2 \geq 1$, so

$\boxed{n_2 \leq 1.283}$.



40. (a) In the diagram the two angles are related by Snell's law:

$n_{\text{air}} \sin \theta_2 = n_{\text{water}} \sin \theta_1$.

Assuming that are both very small, this gives

$n_{\text{air}}\, \theta_2 \approx n_{\text{water}}\, \theta_1$, or

$\theta_2 \approx (n_{\text{water}} / n_{\text{air}})\, \theta_1 = (1.33/1.00)\, \theta_1 = 1.33\, \theta_1$.

It follows that

$d = x/\tan \theta_2 \approx x/\theta_2 \approx x/1.33\, \theta_1$

$= 0.752\, (x/\theta_1) \approx 0.752\, (x/\tan \theta_1)$

$= 0.752\, (4 \text{ m}) = 3 \text{ m}$,

i.e., the light appears to be $\boxed{\text{3 m below the surface of the water}}$.

(b) The frequency of the light depends only on the source and not the medium, so it remains the same in water as in air:

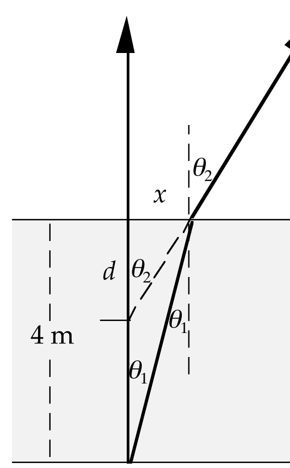
$f = c/\lambda_{\text{air}} = (3.00 \times 10^8 \text{ m/s})/(589 \times 10^{-9} \text{ m}) = \boxed{5.09 \times 10^{14} \text{ Hz}}$.

The wavelength in water is

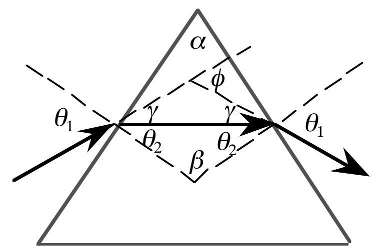
$\lambda_{\text{water}} = \lambda_{\text{air}}\, n_{\text{water}} = (589 \text{ nm})(1.33) = \boxed{783 \text{ nm}}$.

The speed of light in water is

$v_{\text{water}} = c/n_{\text{water}} = (3.00 \times 10^8 \text{ m/s})/1.33 = \boxed{2.26 \times 10^8 \text{ m/s}}$.



41. Refer to the diagram to the right. For minimum deflection, by symmetry the light ray traversing inside the prism is parallel to its base. We have



$\alpha + \beta = 2\theta_2 + \beta = \pi$, which gives

$\theta_2 = \alpha/2$.

The angle of deflection is

$\phi = 2\gamma = 2(\theta_1 - \theta_2)$, or

$\theta_1 = \phi/2 + \theta_2 = \phi/2 + \alpha/2$.

Take the sine of both sides of the last equality above:

$\sin\theta_1 = \sin(\phi/2 + \alpha/2)$.

But from Snell's law we can also find $\sin\theta_1$:

$\sin\theta_1 = n\sin\theta_2 = n\sin(\alpha/2)$.

Equate the two expressions for $\sin\theta_1$ to obtain

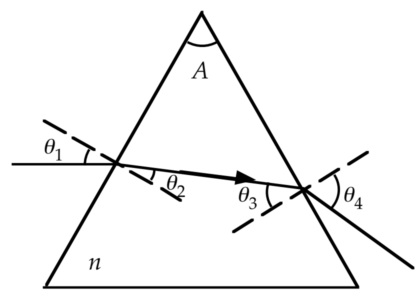
$\sin(\phi/2 + \alpha/2) = n\sin(\alpha/2)$, which we solve for n:

$n = [\sin(\phi/2 + \alpha/2)]/[\sin(\alpha/2)]$

$= [\sin(35°/2 + 60°/2)]/[\sin(60°/2)]$

$= \boxed{1.5}$.

42. We find the apex angle of the triangle from



$\tan(\frac{1}{2}A) = \frac{1}{2}(\text{base})/(\text{height})$

$= \frac{1}{2}(1/2.5) = 0.2$, which gives

$A = 22.6°$.

The angle of incidence is $\frac{1}{2}A$, so we have

$n_{\text{air}}\sin\theta_1 = n\sin\theta_2$;

$(1.00)\sin[\frac{1}{2}(22.6°)] = (1.58)\sin\theta_2$, which gives

$\theta_2 = 7.1°$.

We find the angle of incidence at the second surface from

$\theta_3 = A - \theta_2 = 22.6° - 7.1° = 15.5°$.

For the refraction at the second surface, we have

$n\sin\theta_3 = n_{\text{air}}\sin\theta_4$;

$(1.58)\sin 15.5° = (1.00)\sin\theta_4$, which gives

$\theta_4 = 25.0°$.

Because the normal to the second surface is $\frac{1}{2}A$ above the horizontal, the angle relative to the base of the prism (which is the direction of the incident beam) is

deviation $= \theta_4 - \frac{1}{2}A = 25.0° - \frac{1}{2}(22.6°) = \boxed{13.7°}$.

43. For normal incidence on the boundary between two media the ratio of the intensities of the reflected versus incident wave is

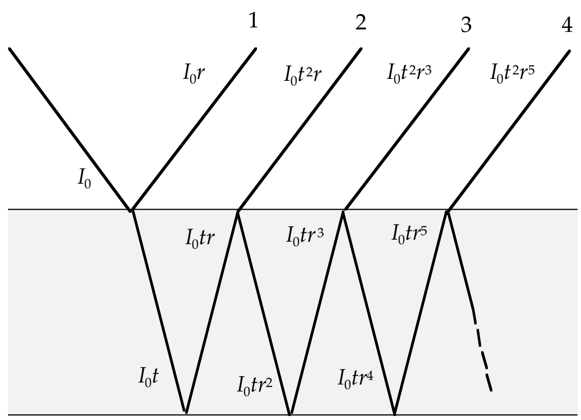
$r = I_r/I_0 = (n_2 - n_1)^2/(n_2 + n_1)^2$

$= (1.5 - 1.0)^2/(1.5 + 1.0)^2 = 0.040$,

meaning that $(1 - 0.040) = 0.96 = 96\%$ of the light is transmitted at each boundary. Each slide has two sides so there are a total of 16 boundaries for eight such slides. The fraction of the intensity of light transmitted is then

$(0.96)^{16} = 0.52 = \boxed{52\%}$.

44.



Similar to the previous problem, for normal incidence on the boundary between two media the ratio of the intensities of the reflected versus incident wave is

$$r = (n_2 - n_1)^2/(n_2 + n_1)^2 .$$

For the first reflection off surface A, $r = (n-1)^2/(n+1)^2$, Thus the intensity of the first reflected beam (beam 1) is

$$I_1 = I_0 r.$$

The fraction of light transmitted into the glass pane is $t = 1 - r$. Compared with beam 1, the second reflected beam (beam 2) has undergone two extra transmissions, so its intensity is

$$I_2 = I_1\, t^2 = I_0\, t^2 r.$$

Thereafter, each successive beam (3, 4, etc.) goes through two extra reflections compared to the previous one, so

$$I_3 = I_2\, r^2 = I_0\, t^2 r^3, \quad I_4 = I_3\, r^2 = I_0\, t^2 r^5, \ \ldots$$

The total intensity of all the reflected beams is then

$$\begin{aligned} I_r &= I_1 + I_2 + I_3 + I_4 + \ldots \\ &= I_0 r + I_0\, t^2 r + I_0\, t^2 r^3 + I_0\, t^2 r^5 + \ldots \\ &= I_0 r + I_0\, t^2 r\,(1 + r^2 + r^4 + \ldots) \\ &= I_0 r + I_0\, t^2 r\,/(1 - r^2) \\ &= I_0 r\,[1 + t^2/(1 - r^2)] = I_0 r\,[1 + (1 - r)^2/(1 - r^2)] \\ &= 2I_0\, r/(1 + r) \\ &= \boxed{2I_0\,(n-1)^2/(n^2+1)}. \end{aligned}$$

For $n = 1.5$, for example, we get $I_r = 2I_0\,(1.5 - 1)^2/(1.5^2 + 1) = 0.15 I_0$. This is certainly finite.

45. The angle of the ray with respect to the normal is the angle of the wave front with respect to the surface. If we let L be the distance along the surface between wave fronts, we have

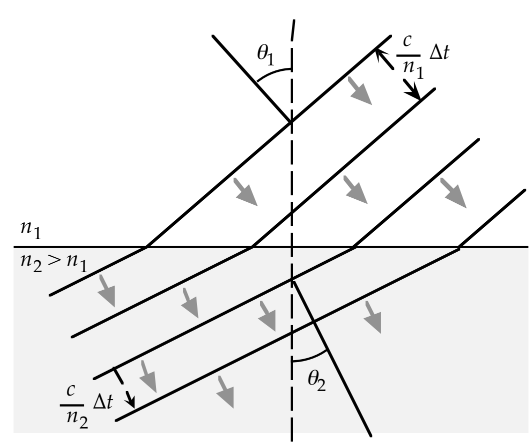
$$\sin\theta_1 = [(c/n_1)\,\Delta t]/L, \quad \text{and}$$

$$\sin\theta_2 = [(c/n_2)\,\Delta t]/L.$$

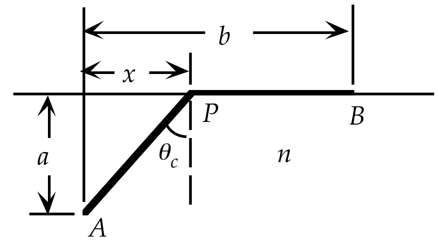
If we divide the two equations, we get

$$(\sin\theta_1)/(\sin\theta_2) = n_2/n_1,$$

which gives $n_1 \sin\theta_1 = n_2 \sin\theta_2$.



46. The ray travels from point A in the medium to point B in air on the surface. Point A is a distance a from the surface. The component of the distance between A and B parallel to the surface is b. If x is the component of the distance between A and the point P, where the ray meets the surface, parallel to the surface, the time of travel is



$$t_{AB} = t_{AP} + t_{PB}$$
$$= (x^2 + a^2)^{1/2}/(c/n) + (b - x)/c.$$

We find the value of x for the minimum time from

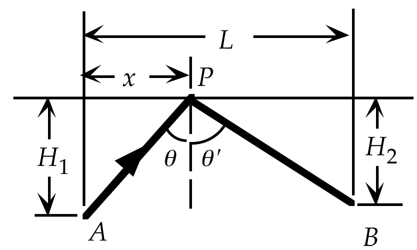
$$dt_{AB}/dx = (n/c)\tfrac{1}{2}(2x)/(x^2 + a^2)^{1/2} - 1/c = 0,$$

which reduces to

$$nx/(x^2 + a^2)^{1/2} = 1.$$

From the diagram, we see that $x/(x^2 + a^2)^{1/2} = \sin\theta_c$, so we have $\sin\theta_c = 1/n$.

47. The ray travels from point A, a distance H_1 from the surface, to point B, a distance H_2 from the surface, after reflecting from the surface. The component of the distance between A and B parallel to the surface is L. If x is the component of the distance between A and the point P, where the ray meets the surface, parallel to the surface, the time of travel is



$$t_{AB} = t_{AP} + t_{PB}$$
$$= (x^2 + H_1^2)^{1/2}/c + [(L - x)^2 + H_2^2]^{1/2}/c.$$

We find the value of x for the minimum time from

$$dt_{AB}/dx = (1/c)\tfrac{1}{2}(2x)/(x^2 + H_1^2)^{1/2} + (1/c)\tfrac{1}{2}(-2)(L - x)/[(L - x)^2 + H_2^2]^{1/2} = 0,$$

which reduces to

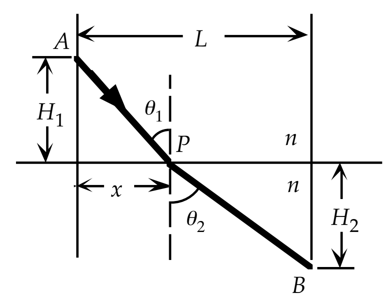
$$x/(x^2 + H_1^2)^{1/2} = (L - x)/[(L - x)^2 + H_2^2]^{1/2}.$$

From the diagram, we see that

$$x/(x^2 + H_1^2)^{1/2} = \sin\theta \text{ and } (L - x)/[(L - x)^2 + H_2^2]^{1/2} = \sin\theta',$$

so we have $\sin\theta = \sin\theta'$, or $\theta = \theta'$.

48. The ray travels from point A in the top medium, a distance H_1 from the surface, to point B in the bottom medium, a distance H_2 from the surface. The component of the distance between A and B parallel to the surface is L. If x is the component of the distance between A and the point P, where the ray meets the surface, parallel to the surface, the time of travel is



$$t_{AB} = t_{AP} + t_{PB}$$
$$= (x^2 + H_1^2)^{1/2}/(c/n) + [(L - x)^2 + H_2^2]^{1/2}/(c/n).$$

We find the value of x for the minimum time from

$$dt_{AB}/dx = (n/c)\tfrac{1}{2}(2x)/(x^2 + H_1^2)^{1/2} + (n/c)\tfrac{1}{2}(-2)(L - x)/[(L - x)^2 + H_2^2]^{1/2} = 0,$$

which reduces to

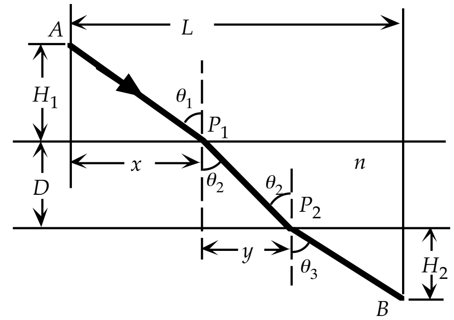
$$x/(x^2 + H_1^2)^{1/2} = (L - x)/[(L - x)^2 + H_2^2]^{1/2}.$$

From the diagram, we see that

$$x/(x^2 + H_1^2)^{1/2} = \sin\theta_1 \text{ and } (L - x)/[(L - x)^2 + H_2^2]^{1/2} = \sin\theta_2,$$

so we have $\sin\theta_1 = \sin\theta_2$, or $\theta_1 = \theta_2$. The beam travels in a straight line.

49. The ray travels from point A in air, a distance H_1 from the top surface, to point B in air, a distance H_2 from the bottom surface. The component of the distance between A and B parallel to the surface is L. If x is the component of the distance between A and the point P_1, parallel to the surface, and y is the component of the distance between P_1 and the point P_2, where the ray leaves the glass, parallel to the surface, the time of travel is



$$t_{AB} = t_{AP1} + t_{P1P2} + t_{P2B}$$
$$= (x^2 + H_1{}^2)^{1/2}/c + (y^2 + D^2)^{1/2}/(c/n) + [(L - x - y)^2 + H_2{}^2]^{1/2}/c.$$

We see that the time depends on the two independent variables, x and y. For the minimum time due to variation in x, we have

$$\partial t_{AB}/\partial x = (1/c)\tfrac{1}{2}(2x)/(x^2 + H_1{}^2)^{1/2} + (1/c)\tfrac{1}{2}(-2)(L - x - y)/[(L - x - y)^2 + H_2{}^2]^{1/2} = 0,$$

which reduces to

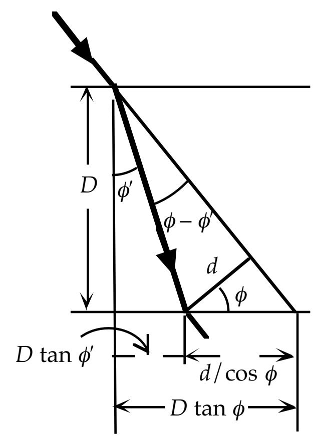
$$x/(x^2 + H_1{}^2)^{1/2} = (L - x - y)/[(L - x - y)^2 + H_2{}^2]^{1/2}.$$

From the diagram, we see that

$$x/(x^2 + H_1{}^2)^{1/2} = \sin\theta_1 \text{ and } (L - x - y)/[(L - x - y)^2 + H_2{}^2]^{1/2} = \sin\theta_3,$$

so we have $\sin\theta_1 = \sin\theta_3$, or $\theta_1 = \theta_3$. The beam emerges parallel to the original direction.

50. We use the analysis from Problem 49, with θ_1 replaced by ϕ and θ_2 replaced by ϕ'. For the minimum time due to variation in y, we have



$$\partial t_{AB}/\partial y = (n/c)\tfrac{1}{2}(2y)/(y^2 + D^2)^{1/2} + (1/c)\tfrac{1}{2}(-2)(L - x - y)/[(L - x - y)^2 + H_2{}^2]^{1/2} = 0,$$

which reduces to

$$ny/(y^2 + D^2)^{1/2} = (L - x - y)/[(L - x - y)^2 + H_2{}^2]^{1/2}.$$

From the diagram, we see that

$$y/(y^2 + D^2)^{1/2} = \sin\phi' \text{ and that}$$
$$(L - x - y)/[(L - x - y)^2 + H_2{}^2]^{1/2} = \sin\phi,$$

so we have $n \sin\phi' = \sin\phi$, or $\phi' = \sin^{-1}[(\sin\phi)/n]$, which is Snell's law.
From the diagram, we see that $D\tan\phi' = (D\tan\phi) - (d/\cos\phi)$, which gives

$$d = D(\tan\phi - \tan\phi')\cos\phi = \boxed{D\left(\sin\phi - \cos\phi\tan\{\sin^{-1}[(\sin\phi)/n]\}\right)}.$$

51. For the violet light, we have

$$n_{\text{violet}}\sin\theta_i = n_{\text{air}}\sin\theta_{\text{violet}};$$
$$(1.3444)\sin 30° = (1.00)\sin\theta_{\text{violet}}, \text{ which gives } \theta_{\text{violet}} = 42.24°.$$

For the red light, we have

$$n_{\text{red}}\sin\theta_i = n_{\text{air}}\sin\theta_{\text{red}};$$
$$(1.3319)\sin 30° = (1.00)\sin\theta_{\text{red}}, \text{ which gives } \theta_{\text{red}} = 41.76°.$$

The angle between the two rays in the air is

$$\Delta\theta = \theta_{\text{violet}} - \theta_{\text{red}} = \boxed{0.48°}.$$

52. From the definition of the index of refraction, we have

$$v = c/n.$$

Because the change in index is small, we differentiate,

$$dv = -(c/n^2)\,dn, \text{ and form the fractional change:}$$
$$dv/v = -dn/n = -[(1.522 - 1.545)/1.545]100 = \boxed{1.5\%}.$$

53. (a) For the refraction from glass to air, we have

$$n \sin \theta = \sin \theta_{air},$$

which gives

$$(\sin \theta)_{max} = 1/n \quad \text{and}$$

$$(\cos^2 \theta)_{min} = 1 - (\sin^2 \theta)_{max} = 1 - 1/n^2 = (n^2 - 1)/n^2 = C/(\omega_0{}^2 - \omega^2).$$

Because $\cos \theta \le 1$, we have

$$\omega_0{}^2 - \omega^2 \ge C, \quad \text{or} \quad \omega^2 \le \omega_0{}^2 - C;$$

$$\omega^2 \le 685 \times 10^{30}\ \text{rad}^2/\text{s}^2 - 529 \times 10^{30}\ \text{rad}^2/\text{s}^2, \text{ which gives } \boxed{\omega \le 12.5 \times 10^{15}\ \text{rad/s}}.$$

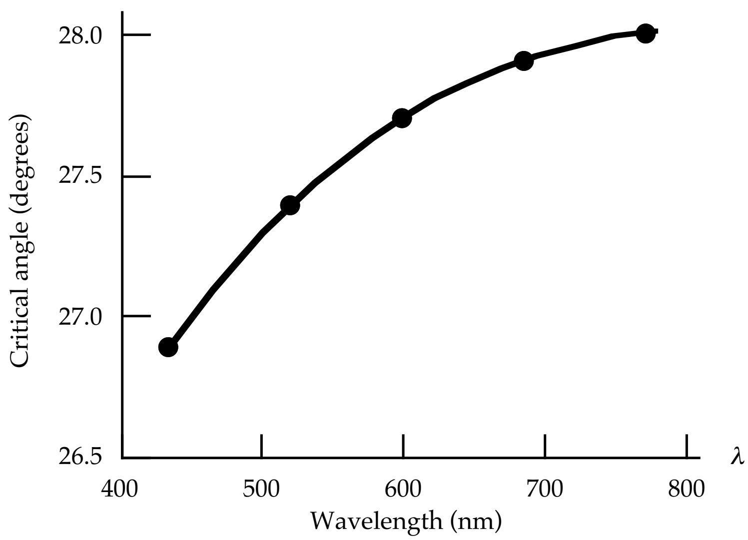
(b) For an angle of 90° in air at this frequency, we have

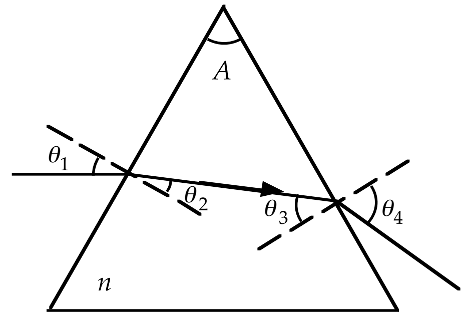
$$(\cos^2 \theta)_{min} = C/(\omega_0{}^2 - \omega_{max}{}^2);$$

$$= (529 \times 10^{30}\ \text{rad}^2/\text{s}^2)/[(685 \times 10^{30}\ \text{rad}^2/\text{s}^2 - (3.2 \times 10^{15}\ \text{rad/s})^2],$$

which gives $\theta = \boxed{27.7°}$.

54.





55. For the refraction at the first surface, we have

$$n_{\text{air}} \sin \theta_1 = n \sin \theta_2;$$
$$(1.00) \sin (\tfrac{1}{2}A) = n \sin \theta_2.$$

We find the angle of incidence at the second surface from

$$\theta_3 = A - \theta_2.$$

For the refraction at the second surface, we have

$$n \sin \theta_3 = n_{\text{air}} \sin \theta_4 = (1.00) \sin \theta_4.$$

If we assume an average index of 1.5 and $A = 60°$, we find

$$\theta_2 = 19.5°,\quad \theta_3 = 40.5°, \text{ and } \quad \theta_4 = 77.0°.$$

Because the changes in angles and indices are small, we approximate them by differentials.

From the three equations, we have

$$0 = \mathrm{d}n \sin \theta_2 + n\cos \theta_2 \,\mathrm{d}\theta_2 ;$$
$$\mathrm{d}\theta_3 = -\mathrm{d}\theta_2;$$
$$\mathrm{d}n \sin \theta_3 + n\cos \theta_3 \,\mathrm{d}\theta_3 = \cos \theta_4 \,\mathrm{d}\theta_4.$$

When we combine these equations, we get

$$\begin{aligned}\cos \theta_4 \,\mathrm{d}\theta_4 &= \mathrm{d}n \sin \theta_3 - n \cos \theta_3 \,\mathrm{d}\theta_2 \\ &= \mathrm{d}n \sin \theta_3 + n\cos \theta_3 \,\mathrm{d}n \sin \theta_2/(n\cos \theta_2) \\ &= \mathrm{d}n\, (\sin \theta_3 \cos \theta_2 + \cos \theta_3 \sin \theta_2)/\cos \theta_2 .\end{aligned}$$

From the sine of the sum of two angles, we have

$$\sin \theta_3 \cos \theta_2 + \cos \theta_3 \sin \theta_2 = \sin(\theta_3 + \theta_2) = \sin A,$$

so our result is

$$\mathrm{d}n = (\cos \theta_4 \cos \theta_2/\sin A)\, \mathrm{d}\theta_4.$$

For a separation of 2°, we have

$$\mathrm{d}n = [(\cos 77.0°)(\cos 19.5°)/(\sin 60°)](2°)(\pi \text{ rad}/180°) = \boxed{0.009}.$$

56. We find the wavelength from

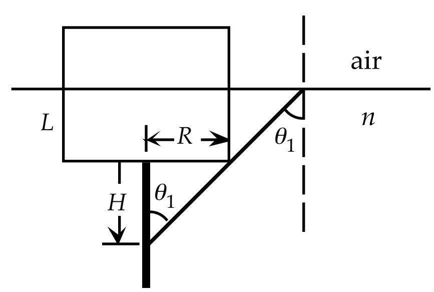
$$\lambda = \lambda_0/n = (450 \text{ nm})/(1.50) = \boxed{300 \text{ nm}}.$$

We find the frequency from

$$f = c/\lambda_0 = (3 \times 10^8 \text{ m/s})/(450 \text{ nm}) = \boxed{6.7 \times 10^{14} \text{ Hz}}.$$

We find the speed from

$$v = c/n = (3 \times 10^8 \text{ m/s})/(1.50) = \boxed{2.0 \times 10^8 \text{ m/s}}.$$



57. We find the critical angle for light leaving the water:

$$n \sin \theta_1 = \sin \theta_2;$$
$$(1.33) \sin \theta_c = \sin 90°, \text{ which gives } \theta_c = 48.8°.$$

We see from the diagram that the highest point on the pin from which light will emerge from the water is determined by a ray that just misses the edge of the cork and reaches the water surface at the critical angle:

$$R/H = \tan \theta_c;$$
$$(1.5 \text{ cm})/H = \tan 48.8°, \text{ which gives}$$
$$H = \boxed{1.31 \text{ cm}}.$$

58. Because the glass surfaces are parallel, the exit beam will be traveling in the same direction as the original beam.
We find the angle inside the glass from

$n_{\text{air}} \sin \theta_1 = n_{\text{glass}} \sin \theta_2$;

$(1.00) \sin 30° = (1.52) \sin \theta_2$, which gives $\theta_2 = 19.2°$.

Relative to the entrance position, we find the distance along the glass where the exit beam leaves from

$D = h \tan \theta_2$.

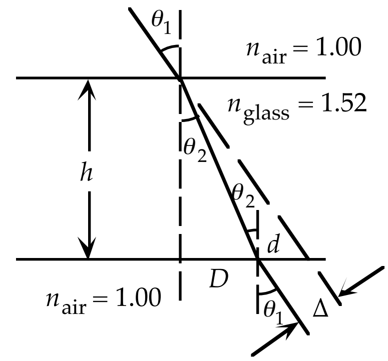
We find the displacement from where the incident beam would have been along the glass surface from

$d = h \tan \theta_1 - h \tan \theta_2 = h(\tan \theta_1 - \tan \theta_2)$.

The perpendicular displacement from the original direction is

$\Delta = d \cos \theta_1 = h(\tan \theta_1 - \tan \theta_2) \cos \theta_1$

$= (2.0 \text{ cm})(\tan 30° - \tan 19.2°) \cos 30° = \boxed{0.40 \text{ cm}}$.



59. For the refraction at the first surface, the angle of incidence is ϕ, so we have

$n_{\text{air}} \sin \theta_1 = n \sin \theta_2$;

$(1.00) \sin \phi = n \sin \theta_2$.

We find the angle of incidence at the second surface from

$\theta_3 = 2\phi - \theta_2$.

For the refraction at the second surface, we have

$n \sin \theta_3 = n_{\text{air}} \sin \theta_4 = (1.00) \sin \theta_4$.

Because the changes in angles and indices are small, we approximate them by differentials. From the three equations,

$0 = \mathrm{d}n \sin \theta_2 + n(\cos \theta_2)\, \mathrm{d}\theta_2$;

$\mathrm{d}\theta_3 = -\mathrm{d}\theta_2$;

$\mathrm{d}n \sin \theta_3 + n(\cos \theta_3\, \mathrm{d}\theta_3 = \cos \theta_4\, \mathrm{d}\theta_4$.

When we combine these equations, we get

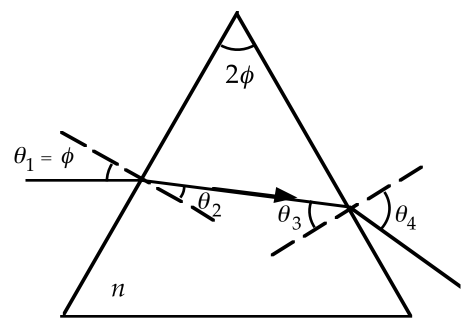
$\cos \theta_4\, \mathrm{d}\theta_4 = \mathrm{d}n \sin \theta_3 - n(\cos \theta_3)\, \mathrm{d}\theta_2 = \mathrm{d}n \sin \theta_3 + n \cos \theta_3\, \mathrm{d}n \sin \theta_2 / (n \cos \theta_2)$

$= \mathrm{d}n\, (\sin \theta_3 \cos \theta_2 + \cos \theta_3 \sin \theta_2) / \cos \theta_2$.

From the sine of the sum of two angles, we have

$\sin \theta_3 \cos \theta_2 + \cos \theta_3 \sin \theta_2 = \sin(\theta_3 + \theta_2) = \sin(2\phi)$, so our result is

$$\boxed{\begin{aligned} &\Delta n = [\cos \theta_4 \cos \theta_2 / \sin(2\phi)]\, \Delta\theta, \text{ where} \\ &\cos \theta_2 = \cos \{\sin^{-1}[(\sin \phi)/n]\} \quad \text{and} \\ &\cos \theta_4 = \cos \left[\sin^{-1}\left(n \sin\{2\phi - \sin^{-1}[(\sin \phi)/n]\}\right)\right]. \end{aligned}}$$



60. The angle of incidence at the surface is 54°. Because $n \propto 1/v$, for the refraction between layers 1 and 2, we have

$(1/v_1) \sin \theta_1 = (1/v_2) \sin \theta_2$, or

$\sin \theta_2 = (v_2/v_1) \sin \theta_1 = (1.05) \sin 54°$,

which gives $\theta_2 = 58.2°$.

For the refraction between layers 2 and 3, we have

$(1/v_2) \sin \theta_2 = (1/v_3) \sin \theta_3$, or

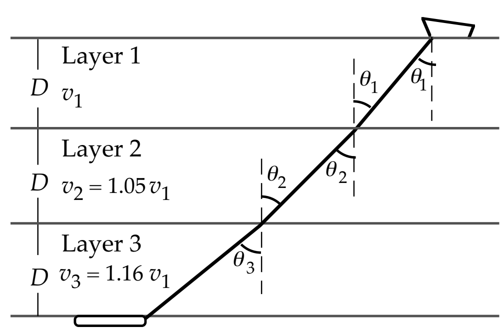
$\sin \theta_3 = (v_3/v_2) \sin \theta_2 = (1.16/1.05) \sin 58.2°$,

which gives $\theta_3 = 69.8°$.

The total horizontal distance is

$L = D \tan \theta_1 + D \tan \theta_2 + D \tan \theta_3$

$= (60 \text{ m}) \tan 54° + (60 \text{ m}) \tan 58.2° + (60 \text{ m}) \tan 69.8° = \boxed{342 \text{ m}}$.



61. For the refraction at the first surface, we have

$$\sin \theta_i = n \sin \theta_2 .$$

We find the angle of incidence at the second surface from

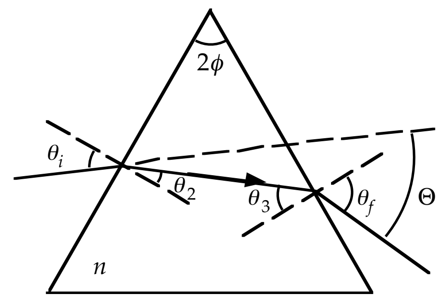
$$\theta_3 = 2\phi - \theta_2 .$$

For the refraction at the second surface, we have

$$n \sin \theta_3 = \sin \theta_f .$$

The total deflection angle is the sum of the deflections that take place at each surface:

$$\Theta = (\theta_f - \theta_3) + (\theta_i - \theta_2) = \theta_i + \theta_f - 2\phi$$
$$= \sin^{-1}(n \sin \theta_2) + \sin^{-1}[n \sin(2\phi - \theta_2)] - 2\phi.$$



This gives the deflection as a function of θ_2 . To find the angle θ_2 that minimizes the deflection, we set $d\Theta/d\theta_2 = 0$:

$$\frac{d\Theta}{d\theta_2} = \frac{n \cos \theta_2}{\sqrt{1 - (n \sin \theta_2)^2}} - \frac{n \cos(2\phi - \theta_2)}{\sqrt{1 - \left[n \sin(2\phi - \theta_2)\right]^2}} + 0 = 0, \quad \text{or}$$

$$\frac{n \cos \theta_2}{\sqrt{1 - (n \sin \theta_2)^2}} = \frac{n \cos(2\phi - \theta_2)}{\sqrt{1 - \left[n \sin(2\phi - \theta_2)\right]^2}} .$$

We can see by inspection that

$$\theta_2 = 2\phi - \theta_2 , \quad \text{or} \quad \theta_2 = \phi.$$

From the first refraction equation, we have

$$\sin \theta_i = n \sin \theta_2 = n \sin \phi, \quad \text{or} \quad \theta_i = \boxed{\sin^{-1}(n \sin \phi)}.$$

Note that, for minimum deflection, the ray goes through the prism symmetrically.

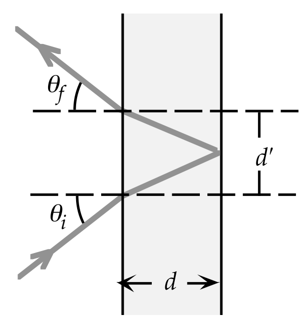
62. From the symmetry of the reflection at the back surface, for the two refractions, we have

$$\sin \theta_i = n \sin \theta_2;$$
$$n \sin \theta_2 = \sin \theta_f ,$$

which gives $\theta_f = \theta_i$ and $\sin \theta_2 = (\sin \theta_i)/n.$

The displacement along the glass is

$$d' = 2d \tan \theta_2 = 2d (\sin \theta_2)/(\cos \theta_2) = 2d (\sin \theta_2)/(1 - \sin^2 \theta_2)^{1/2}$$
$$= 2d [(\sin \theta_i)/n]/[1 - (\sin^2 \theta_i/n^2)]^{1/2} = \boxed{2d (\sin \theta_i)/(n^2 - \sin^2 \theta_i)^{1/2}}.$$



63. (a) We find the distance the earth moves during a period of Io from

$$d = v_{\text{earth}}T$$
$$= (30 \text{ km/s})(42.5 \text{ h})(3600 \text{ s/h}) = \boxed{4.6 \times 10^6 \text{ km}}.$$

(b) When Earth is moving toward Jupiter, the light that indicates the end of the period travels a shorter distance than the light that indicated the start of the period, by the distance that Earth has moved. This means that the measured period is smaller. We find the speed of light from

$$c = d/\Delta t = (4.6 \times 10^9 \text{ m})/(15 \text{ s}) = \boxed{3.1 \times 10^8 \text{ m/s}}.$$